

NEW CORRECTION PROCEDURES FOR THE FAST FIELD PROGRAM WHICH
EXTEND ITS RANGE

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SUMMARY

An FFP algorithm has been developed based on the method of Lee et al* for the prediction of sound pressure level from low frequency high intensity sources. In order to permit accurate predictions at distances greater than 2km, new correction procedures have had to be included in the algorithm. Certain functions, whose Hankel transforms can be determined analytically, are subtracted from the depth dependent Green's function. The distance response is then obtained as the sum of these transforms and the FFT of the residual k dependent function. One procedure, which permits the elimination of most complex exponentials, has allowed significant changes in the structure of the FFP algorithm, which has resulted in a substantial reduction in computation time.

1. INTRODUCTION

Sound pressure levels at large distances from a point source close to the ground have been predicted using ray based procedures¹ in enhanced zones and residue calculations² in strong shadow zones. In the published literature (see references 1 and 2) these predictions have been shown to be approximately valid. The errors for the predictions in the enhanced zone increase when ground reflections become important and when landing ray densities become small. In the shadow zone errors increase when the sound speed gradient becomes small. Both the above procedures are inaccurate or inoperable in the transition regions between shadow and enhancement.

Since the publication of the first paper on the FFP³ for atmospheric sound propagation this method has been increasingly used for sound pressure level prediction. This has largely occurred because the FFP can operate irrespective of whether there are shadow or enhanced or even mixed conditions present. Moreover the FFP can take proper account of ground reflection.

The most widely known FFP algorithm, the CERL-FFP, stems from the method of Lee et al⁴, which was a development of the algorithm described in Raspet et al's first paper³.

Starting from reference 4 we have reworked the analysis to enable us to produce our own FFP algorithm, structured in such a way that we could incorporate a variety of corrections in k space and thereby extend the range of validity of the result in the transformed (real) space.

* Lee et al. J. Acoust. Soc. Am., 79, 1986, pp 628-634.

In section 2 of this paper a brief description is given of the development of our first prototype algorithm. A survey of previously published k spectral corrections applied to the first prototype is given in section 3. Section 4 describes our second algorithm which is based on a ' γ averaging' procedure and section 5 describes a technique adopted for speeding up the computation. In the last section, 7, a comparison between the FFP and our other model's predictions is given for a realistic case.

2. DEVELOPMENT OF THE FIRST FFP ALGORITHM

2.1 Sign Convention

Lee et al⁴ replaced the system of atmospheric strata by an analogue electrical network. In our reworking of the model we found that this was not necessary and that retention of acoustic equations for pressure and particle velocity ensured greater clarity. In addition our analysis showed that great care had to be exercised with the signs used in the ladder calculation.

Raspet³ correctly drew attention to the need for different signs for the characteristic admittance dependent on the direction of the particle velocity, which gives the correct sign for the characteristic admittance of top stratum, $Y_{CO} = + i\gamma_0/\omega\rho_0$ where γ_0 is the propagation constant and ρ_0 is the density and for the characteristic admittance of the ground $Y_{CM} = + 1/Z$ where Z is the usual ground impedance.

None of the published papers on the FFP, including the most recent ones (see Franke et al⁵), make it clear that the signs used in the ladder admittance calculation must be chosen in accordance with the direction in which the calculation is performed. The equation for calculating an admittance (Y_{new}) at the stratum interface nearer to the source from the admittance at the stratum's other interface (Y_{old}) is

$$Y_{new} = Y_{cm} \frac{\epsilon \tanh \gamma_m \ell_m + Y_{old}/Y_{cm}}{1 + \epsilon (Y_{old}/Y_{cm}) \tanh \gamma_m \ell_m} \quad (1)$$

where the characteristic admittance for layer m , is $Y_{cm} = \epsilon i\gamma_m/\omega\rho_m$, γ_m being the propagation constant, ρ_m the density and ℓ_m the stratum thickness; ϵ is +1 when working upwards and -1 when working downwards. We note that the ϵ 's cancel in (1) so that the same equation can be used whether working above or below the source interface. However in view of ϵ in the top semi-infinite layer being set to +1 the admittance calculated just above the source interface, $Y(z_s)^-$, has opposite sign to that calculated just below that height, $Y(z_s)^+$.

The z dependent Green's function at the source, $P(z_s)$, is obtained from the known discontinuity in particle velocity at that location. Using our sign convention and noting the opposite signs of $Y(z_s)^+$ and $Y(z_s)^-$ a negative sign must appear in the denominator of $P(z_s)$.

$$P(z_s) = \frac{-(2i/\omega\rho_s)}{Y(z_s)^+ - Y(z_s)^-} \quad (2)$$

Likewise performing the calculation of the Green's function at the detector, $P(z_D)^*$ and retaining the above convention for ϵ , the equation for calculation of p at the stratum interface nearest the source (P_{new}) from that at that stratum's other interface (P_{old}) becomes

$$P_{\text{new}} = P_{\text{old}} \left\{ \cosh \gamma_m \ell_m + \epsilon \sinh \gamma_m \ell_m \left[\frac{Y_{\text{old}}}{Y_{\text{cm}}} \right] \right\} \quad (3)$$

irrespective of whether the receiver is above or below the source.

2.2 Ground Impedance

The early FFP algorithms used the Delany Bazley⁶ model for the ground impedance. Attenborough has suggested that his four parameter model⁷ be used instead since this gives much smaller and more realistic impedance values at the low frequencies of interest in this study.

2.3 Damping Coefficient

The k spectrum calculated with the above ladder procedure has a large spike at the k value closest to ω/c_0 where c_0 is the sound speed for the top semi-infinite top layer and also a number of subsidiary spikes at k values nearest to ω/c_m . These infinities produce errors when the Fourier transform is performed. A global damping is usually introduced, preferably by subtracting a small imaginary quantity $i\mu$ from k , where μ is typically 10^{-4} . The transform is corrected for the effect of the damping by multiplying it by $e^{\mu x}$

This procedure effectively produces different damping effects on each spike dependent on their proximity to the nearest k sample point. The k sampling errors are therefore only partially removed.

3. APPLICATION OF PUBLISHED k SPECTRAL CORRECTIONS

3.1 Candel and Crance's k Shift Procedure

Candel and Crance⁸ proposed a method which ensured the k sample intervals were chosen so that the main peaks all occurred well away from the sample points. Thereby the transform errors arising from the presence of spikes in the k spectrum were substantially reduced.

We set up an algorithm to readjust the k sampling interval, Δk , until the 5 main spikes were more than a limiting distance ($\Delta k/10$) from the nearest k sample point. The method requires a full ladder calculation for typically 1/8th of the total k range centred on $k_0 = \omega/c_0$ for each setting of Δk . This can be time consuming when large numbers of strata are considered.

The method worked well for large Δk values when the total number of k samples, N , was less than $1k$. For the larger N values required for ranges greater than $2km$ it became increasingly difficult to ensure the main spikes were distanced more than the above limiting value from the sample points.

* The term k spectrum in this paper refers to $F = k P(z_D)$ which is the function to be Fourier transformed.

The k spectrum very often has a non zero asymptotic value or tail as k goes to infinity. This tail invariably occurs when source and receiver are approximately the same height above ground.

Introduction of a cut-off for the spectrum at a value k_{\max} produces large ripples in the transform which increase with range and with the proximity of k_{\max} to ω/c_0 .³ In this study we set k_{\max} at a value where the change in F was less than a prescribed limit, but this remained unsatisfactory.

The k spectral tail can be removed by subtracting the function $g(k)$ given by Richards and Attenborough⁹ from $F(k)$.

$$g(k) = (A + e^{-kz_s}) (1 - e^{-\delta k}) \quad (4)$$

where z_s is the source height and $A = |F(k_{\max})|$ and δ is the derivative of $|F(k)|$ at small k . $g(k)$ has an exact Hankel transform which is added to the Fourier transform of $F(k) - g(k)$. The above tail remover does not have the correct phase at small k and this produces a small error in the transform.

Experiments with functions, with exact Hankel transforms which mimic the $F(k)$ behaviour at small k and near the main spike are in progress.

In Figure 1 the attenuation for a source and receiver 2m above an impedance ground at 50Hz is shown with and without the tail remover.

4. MODIFIED FFP ALGORITHM USING ' γ ' AVERAGING¹

For the large ranges of interest in this study very small Δk values must be used therefore the Candel and Crance method does not work. A novel technique for obtaining a k spectrum which can be more reliably transformed has been developed.

The γ averaging procedure is most easily understood by considering a single step in the admittance calculation for one stratum, index m , as described in section 2.

Writing equation (1) in matrix form

$$\begin{bmatrix} P \\ U \end{bmatrix}_{\text{new}} = M_m \begin{bmatrix} P \\ U \end{bmatrix}_{\text{old}} \quad (5)$$

where the matrix M_m has elements which are functions of k and are usually evaluated at a single k sample value, say k_r .

$$M_m = \begin{bmatrix} \cosh \gamma_m \ell_m & \epsilon Z_{cm} \sinh \gamma_m \ell_m \\ \frac{\epsilon \sinh \gamma_m \ell_m}{Z_{cm}} & \cosh \gamma_m \ell_m \end{bmatrix} \quad (6)$$

$$\text{where } \gamma_m = \sqrt{k_r^2 - \omega^2/c_m^2}$$

For Fourier integrals of smoothly varying functions $F(k)$ (which have to be approximated by a discrete Fourier transform), the optimal sampling is at equidistant k_r values. These $F(k)$ samples can be assumed to approximate the contribution to the integral over the interval $k_r - \Delta k/2$ to $k_r + \Delta k/2$. In the presence of integrable infinities a different averaging process must be used.

The simplest method follows from changing the integration variable in the above interval from k to γ ; then elementary averaging leads to (6) with $\gamma_m(k_r)$ replaced by

$$\bar{\gamma}_m = [\gamma_m(k_r - \Delta r/2) + \gamma_m(k_r + \Delta r/2)]/2$$

We employ this averaging method for those intervals where γ^2 changes sign.

Provided c_m is real (no damping) the values of γ_m in the integrand are either pure imaginary or pure real and so are the γ_m values at all sample points except those where the above averaging is employed. Hence the hyperbolic functions in (6) can be replaced by real trigonometric or hyperbolic functions, the full complex functions being required at the above critical points only. A speed up by a factor of 3 was obtained by this procedure over that using (6) in its general complex form. This high speed algorithm cannot be used if artificial damping is present.

5. k SPACE INTERPOLATION

In order to obtain predictions out to large distances the number of k samples, N , for a given k_{\max} must be increased. As the major part of the computation lies in the determination of the z dependent kernel at each k value, we can achieve considerable improvements in calculation speed by interpolating $F(k)$ where it varies smoothly. This applies to all regions of the spectrum lying outside the range spanned by ω/c_m for all layers (typically $k_{\max}/8$). It was found that $F(k)$ only needed to be evaluated every 8th point in the smooth region.

For most cases linear interpolation of $F(k)$ is adequate. However at very large ranges fluctuations occur in the transform due to the small discontinuities in the slope of the interpolated $F(k)$. These errors can be reduced by using cubic interpolation.

6. STRATUM QUANTISATION ERROR

In Figure 2 the predicted attenuation above an impedance ground is shown for two different stratum configurations with the same small linear sound speed gradient (0.01 s^{-1}). For the solid curve all strata above 30m are taken as 27m thick and for the dotted curve as 54m thick. It is clear that the undulations in the dotted curve are much bigger than for the solid curve. This occurs because the FFP uses mid stratum sound speed averages for the whole of each stratum. This produces a stratum sampling error which gets worse the thicker the strata and the larger the sound speed gradient. There are two remedies: one is to use many very fine strata that increases the amount of computation, the other is to use linear sound speed variations in each stratum and use a modified procedure employing Airy functions¹⁰.

7. COMPARISON OF THE FFP PREDICTIONS WITH RAY/RESIDUE MODEL PREDICTIONS

In Figure 3 the attenuation predicted by the FFP for a lapse-inversion-lapse meteorology is compared with that obtained by a hybrid method. This latter method combines the predictions from a ray based model for the enhanced regions with

those from a residue model for the shadow regions. The advantages of using the FFP are apparent in this figure. The residue model overpredicts the shadow attenuation and the ray model is restricted to giving predictions only in the region where there are ray landing points. Neither of these models can properly deal with the transitional region between shadow and enhancement.

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Effect of Tail Remover Function

$\theta=0^\circ$, $\sigma=120$, $z_s=2.0$, $z_b=2.0$

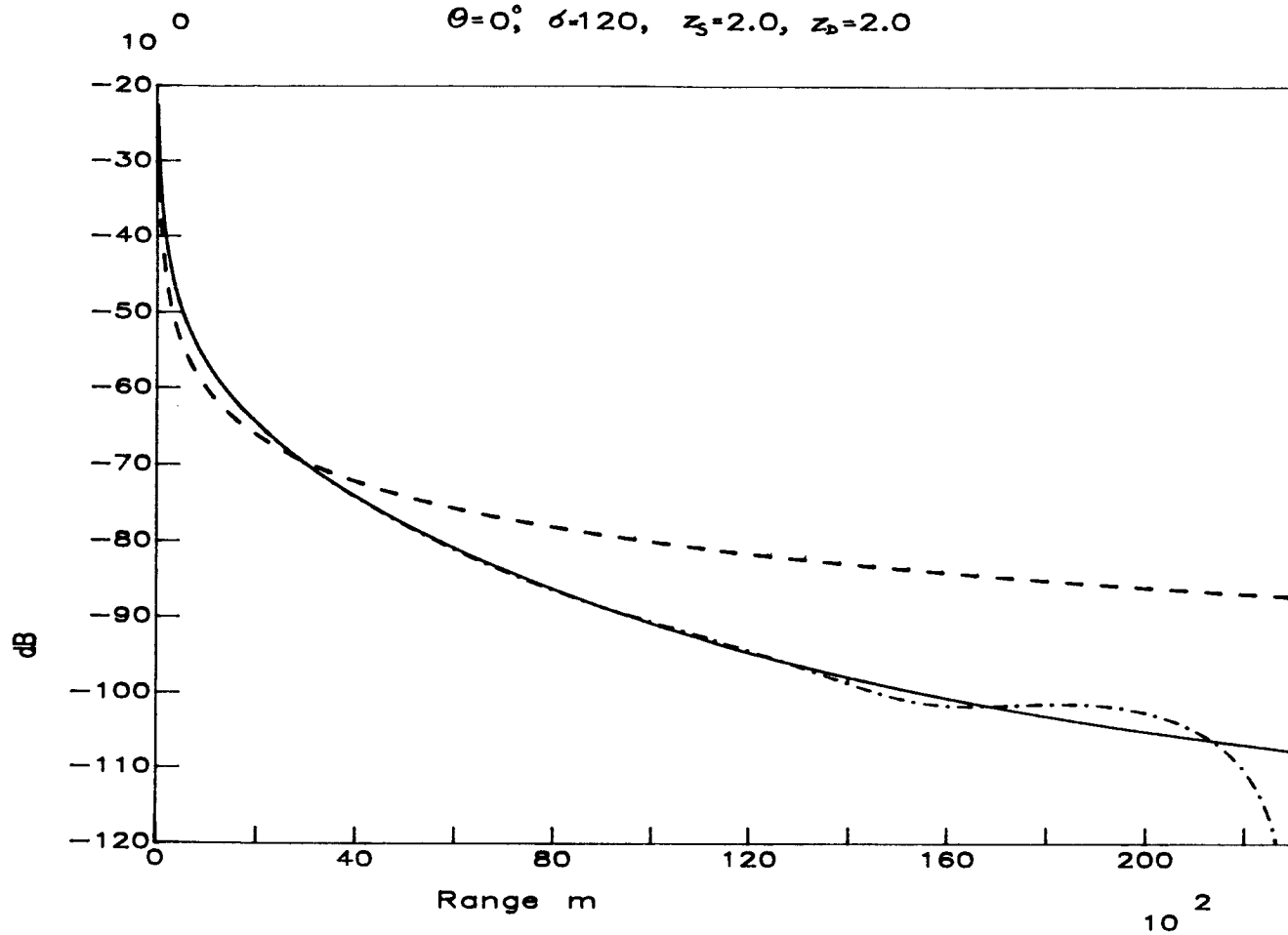


Figure 1

Effect of the Richards-Attenborough Tail Remover Function.
Still air and an impedance ground.

- | | | |
|--------------------|---|----------------------|
| Dashed line | - | free field |
| Solid line | - | with tail remover |
| Dashed-dotted line | - | without tail remover |

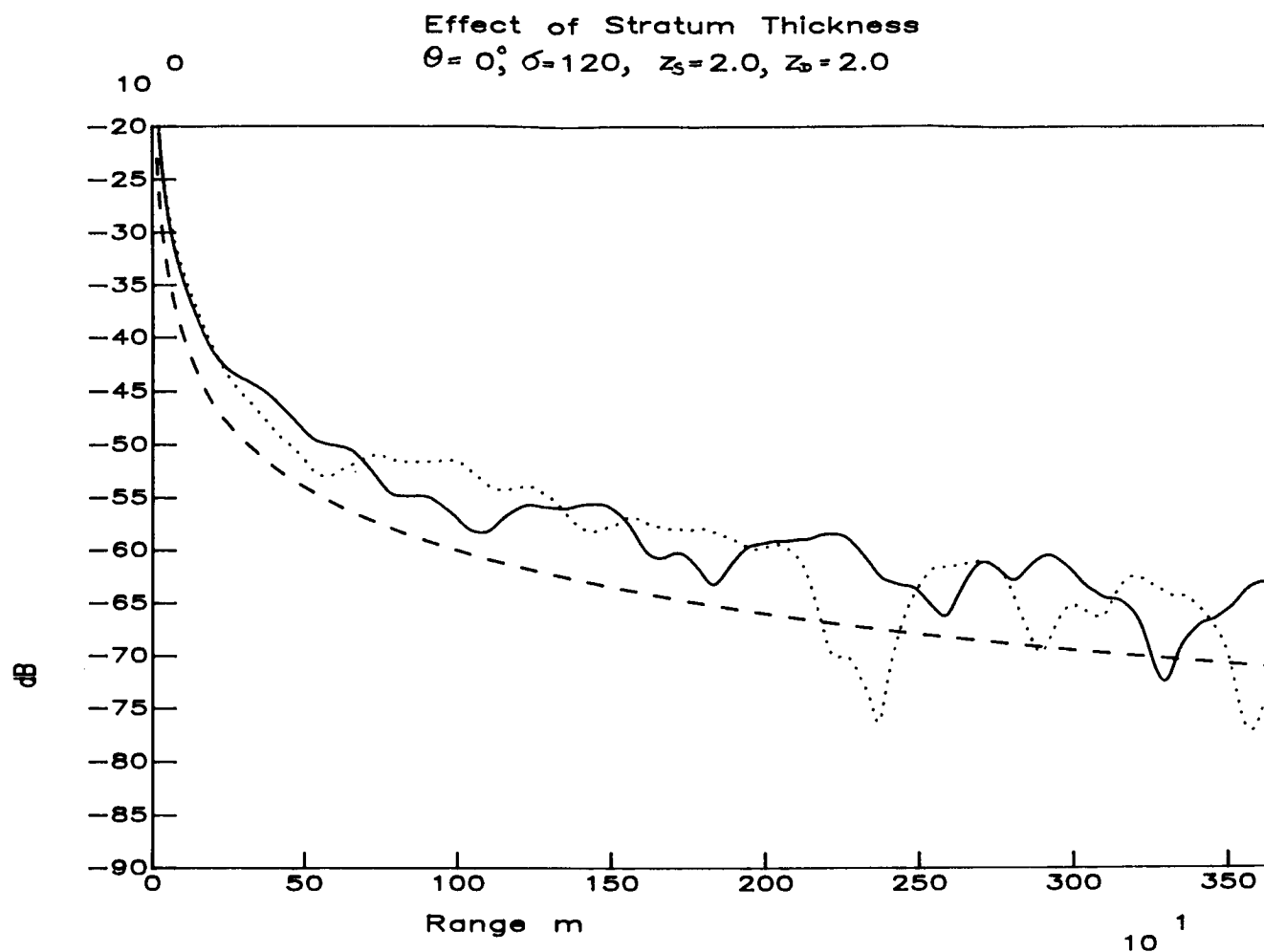


Figure 2 Effect of Stratum Thickness
 Sound speed gradient 0.01 and an impedance ground.

Dashed line	-	free field
Solid line	-	27m thick strata above 30m
Dotted line	-	54m thick strata above 30m

Comparison with Hybrid Model Predictions

$\theta = 0^\circ$, $\sigma = 120$, $z_s = 2.0$, $z_p = 2.0$

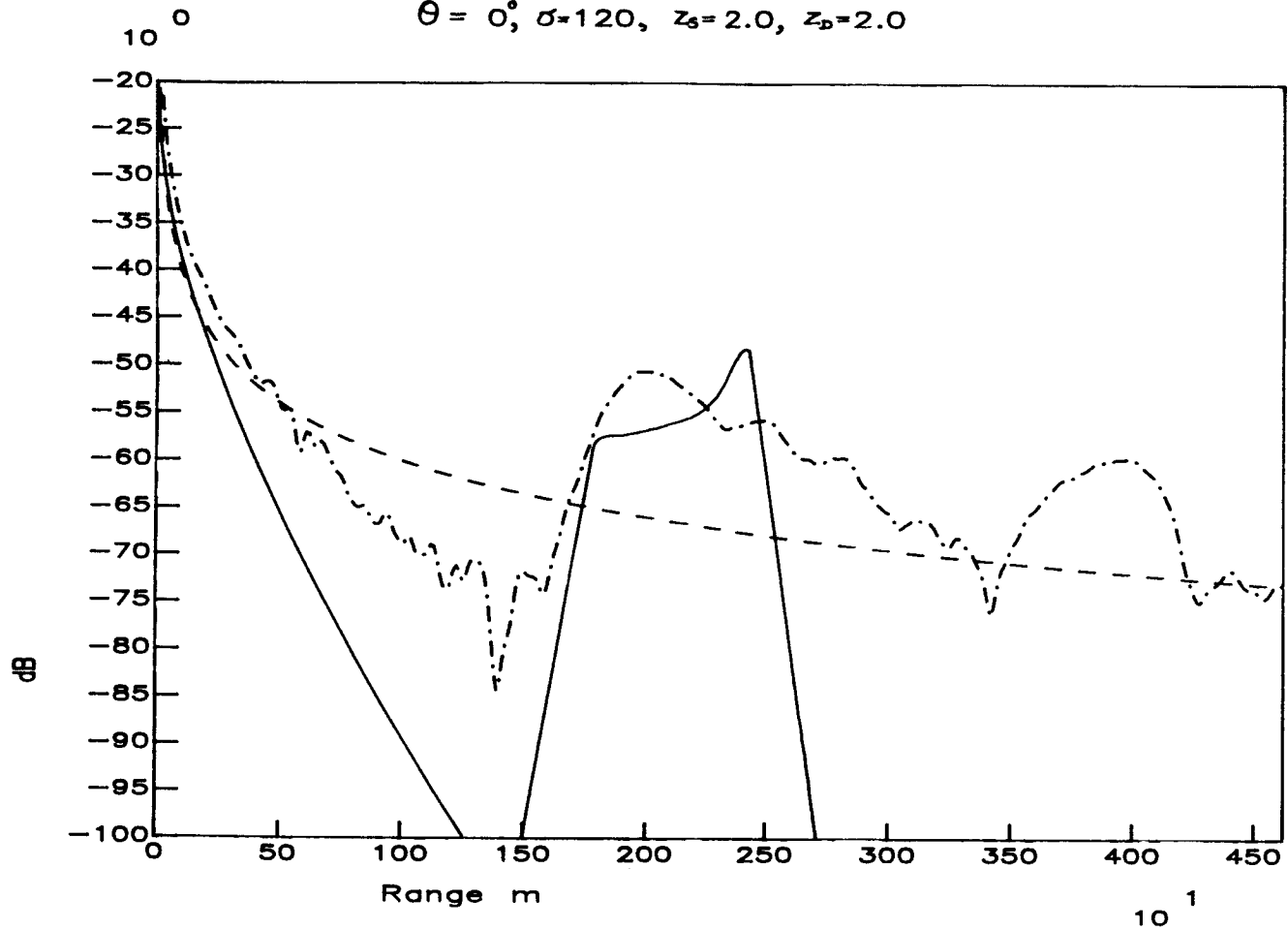


Figure 3

Comparison of FFP Predictions with those from a Hybrid Model. Sound speed gradient up to 100m -0.1, from 100m to 200m +0.1 and above 200m -0.05 and an impedance ground.

- Dashed line - free field
- Solid line - hybrid model prediction
- Dashed-dotted line - FFP prediction